Improper Integrals and the Integral Test

$$\int_{-\infty}^{\infty} \frac{1}{x^2+1}\,dx = \lim_{a\to-\infty}\lim_{b\to\infty}\int_a^b \frac{1}{x^2+1}\,dx$$

$$= \lim_{a\to-\infty}\lim_{b\to\infty}\left(\tan^{-1}x\right)\Big|_a^b$$

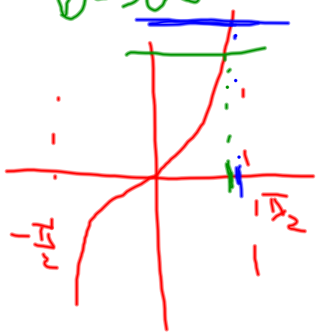
$$= \lim_{a\to-\infty}\lim_{b\to\infty} \tan^{-1}b - \tan^{-1}a$$

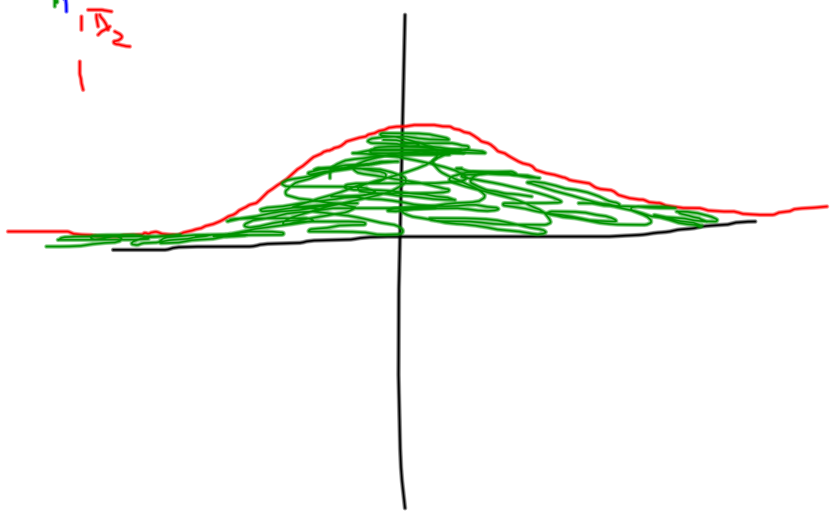
$$\frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi$$

$$\lim_{b\to\infty} \tan^{-1}b = \frac{\pi}{2}$$

$\tan : \frac{\sin}{\cos}$ $\to \infty$, $\to 0$

$$\lim_{a\to-\infty} \tan^{-1}a = -\frac{\pi}{2}$$





$\int u^{-2}\,du$

$\frac{u^{-1}}{-1}$

$$\int_0^2 \frac{1}{(x-1)^2}\,dx$$

$$= \lim_{a\to 1^-}\int_0^a \frac{1}{(x-1)^2}\,dx + \lim_{b\to 1^+}\int_b^2 \frac{1}{(x-1)^2}\,dx$$

find an antiderivative

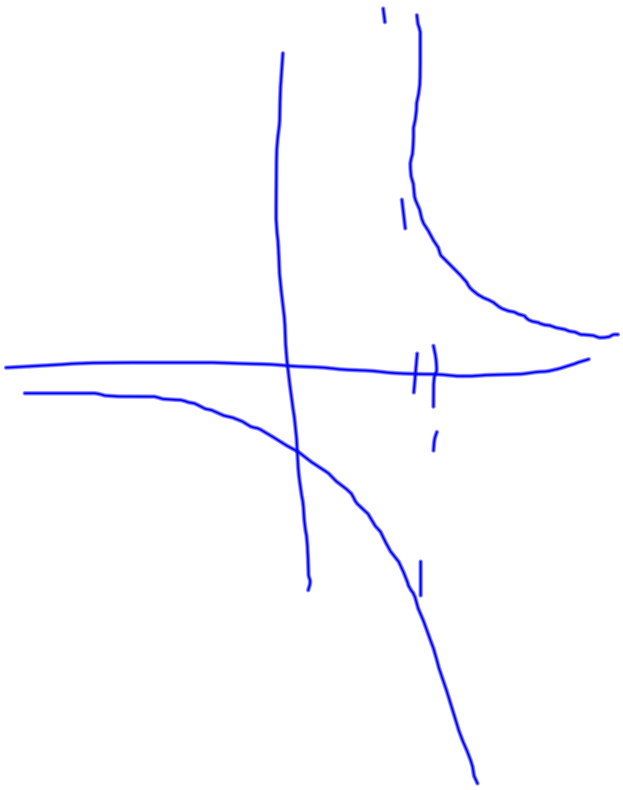
$$\frac{-1}{(x-1)}\Big|_0^2$$

$$= \frac{-1}{(1)} - \left(\frac{-1}{-1}\right) = -2$$

$$= \lim_{a\to 1^-}\left[\frac{-1}{(a-1)} - \frac{-1}{(-1)}\right]$$

$$\lim_{a\to 1^-}\frac{-1}{(a-1)}$$

$$\frac{1}{x-1}$$



Harmonic Series

$$\sum_{k=1}^{\infty} \frac{1}{k} = \frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots$$

diverges

$$\int_1^{\infty} \frac{1}{x}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{x}\,dx = \lim_{b\to\infty} \ln b - \ln 1 = \infty$$

$$\sum \frac{1}{x^2}$$

$$1+\int_1^{\infty} \frac{1}{x^2}\,dx = \lim_{b\to\infty}\left(-\frac{1}{x}\right)\Big|_1^b = \lim_{b\to\infty} -\frac{1}{b}-\left(-\frac{1}{1}\right) = 1$$

$$\int_{-\infty}^{\infty} \frac{1}{x^2+1}\,dx = \lim_{b\to\infty}\lim_{a\to-\infty}\int_a^b \frac{1}{x^2+1}\,dx$$

$$= \lim_{b\to\infty}\lim_{a\to-\infty}\left(\tan^{-1}x\right)\Big|_a^b$$

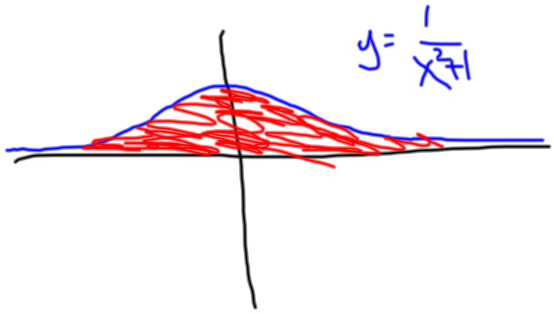
$$= \lim_{b\to\infty}\lim_{a\to-\infty} \tan^{-1}b - \tan^{-1}a = \frac{\pi}{2} - \left(-\frac{\pi}{2}\right)$$

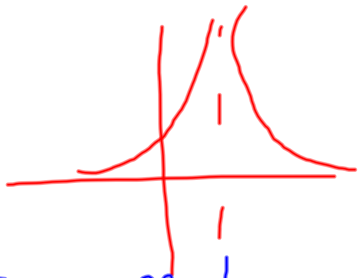
$$= \pi$$

$$\lim_{b\to\infty} \tan^{-1}b = \frac{\pi}{2}$$

$$\lim_{a\to-\infty} \tan^{-1}a = -\frac{\pi}{2}$$

$$\frac{\sin x}{\cos x}$$





$$\int_0^2 \frac{1}{(x-1)^2}\,dx = \lim_{a\to 1^-}\int_0^a \frac{1}{(x-1)^2}\,dx + \lim_{b\to 1^+}\int_b^2 \frac{1}{(x-1)^2}\,dx$$

$$\left(-\frac{1}{x-1}\right)\Bigg|_0^2 = -\frac{1}{1} - \left(-\frac{1}{-1}\right) = -1-(+1) = -2$$

$$\lim_{a\to 1^-}\int_0^a \frac{1}{(x-1)^2}\,dx = \lim_{a\to 1^-}\left(-\frac{1}{x-1}\right)\Bigg|_0^a$$

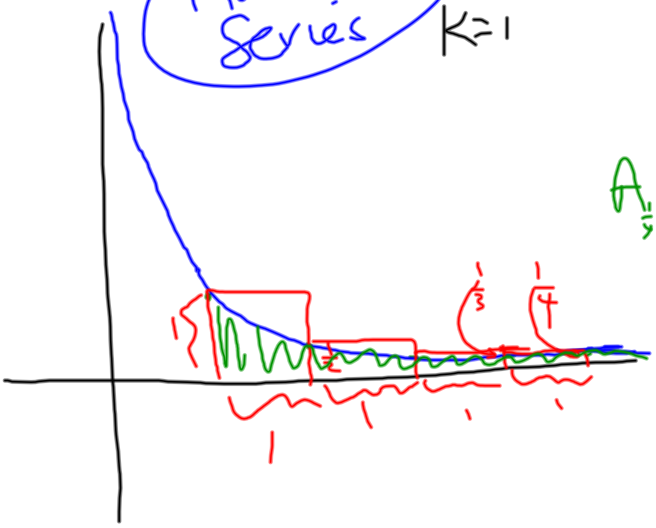
$$= \lim_{a\to 1^-} \frac{-1}{(a-1)} + \frac{1}{0-1} = \lim_{a\to 1^-} -1 - \frac{1}{a-1}$$

$$= -1-\infty$$

Harmonic Series

$$\sum_{K=1}^{\infty} \frac{1}{K} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$$

DIVERGES



$$A_{\frac{1}{x}} \quad \int_1^{\infty} \frac{1}{x}\,dx$$

$$= \lim_{b\to\infty} \int_1^{b} \frac{1}{x}\,dx$$

$$= \lim_{b\to\infty} (\ln b - \ln 1)$$

$$= \infty$$

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